AN EMPIRICAL INVESTIGATION OF THE MARKOWITZ AND SHARPE'S SINGLE INDEX MODEL IN BUILDING OPTIMUM PORTFOLIO

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1.   INTRODUCTION

A portfolio is a mix of diverse investment assets that are mixed and matched to achieve an investor's goal. A portfolio is a professional way to present projects that keeps them from getting lost. The key to build an optimum portfolio is to achieve the desired goal with the given resources. Picking a low-risk stock in the expectation of high profits is the optimum investment. Great risk means high reward in the investment game, and vice versa. According to Harry Markowitz's Modern Portfolio Theory, potential risk may be diffused throughout a portfolio through diversification. This implies investing in multiple industries and balancing a high-risk investment with a low-risk stock. Building a portfolio needs time, thought and organization, and is not as simple as putting random pieces of work into a folder. Building a stock portfolio is challenging since it involves extensive research. But, in the end, the stock market's potential to grow through time will almost certainly lead to the achievement of objectives. People viewing portfolios will be able to recognise abilities immediately and will be more likely to be impressed by what they see. In 1952, Markowitz developed a model for risk-averse investors to create an optimum asset portfolio. Many academics and scholars have drawn inspiration from Markowitz's pioneering work, Portfolio Selection, and subsequent investigations. His methodology was designed to be practical and easy to apply. However, it is ironic that the volume of work required to construct an Optimum portfolio was staggering, and thus the model was rarely used in practice (Elton, Gruber, and Padberg, 1976). Researchers have developed and simplified the portfolio development method in response to these issues. William F Sharpe, devised a simplified alternative to Markowitz's model called Sharpe's single index model that significantly reduces data input and computational requirements. It is the popular that is more than half a century old method is still considered one of the most fundamental and widely used models. The purpose of this research is to construct and compare an Optimum portfolio between Markowitz and Sharpe's single index model for the stocks listed from the Dow Jones

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Industrial Average Index (DJIA index). The study is relevant in present times since only few studies have constructed an optimum portfolio that compares the Markowitz model to Sharpe's Single Index Model (SIM) during the period considered for study.

2.   OBJECTIVES OF THE STUDY

To choose securities from DJIA index based on selected parameters namely; Net Profit Margin (NPM), Return on Equity (ROE), Liquidity Ratio (LR), Interest Coverage Ratio (ICR), and Assets Turnover Ratio (ATOR).

To compare optimum portfolio based on Harry Markowitz model and Sharpe's Single index model.

3.   DATA AND METHODOLOGY

The study employs descriptive and secondary data by using DJIA index as a market index. Various parameters such as NPM, ROE, LR, ICR, and ATOR are taken into account while picking stocks from the DJIA index. The data for these financial ratios was collected for five-year period from sources such as macrotrends.net and gurufocus.com, and weighted averages were calculated for each parameter. While calculating weighted averages, recent years are given more weightages and distant years are given the least weightages. For the selection of securities, companies that meet the criteria of NPM greater than or equal to 10%, ROE greater than or equal to 10%, LR greater than or equal to 1, ICR greater than or equal to 10%, and ATOR greater than or equal to 0.50 are selected. Annual adjusted closing prices of selected individual stocks and indexes were obtained from yahoofinance.com over the period of 30 years, from 1993 to 2022. 365 days T-bill rate is taken as a proxy of risk-free rate of return.

4.   SAMPLE COMPANIES

The following table shows various selection parameters and selected securities from the DJIA index for constructing an optimum portfolio.

Table 1: Shows the list of companies selected from DJIA index

Sl. No.	Company Name	Weightage Average					Remarks
		NPM	ROE	LR	ICR	ATOR	
1	MMM 3M	16.11	319.79	1.36	15.48	0.75	Selected
2	AXP American Express	14.89	26.75	1.72	4.41	0.20	Rejected
3	AAPL Apple	22.84	95.48	1.19	28.14	0.87	Selected
4	BA Boeing	-5.88	401.86	0.36	7.77	0.54	Rejected
5	CAT Caterpillar	9.95	32.83	1.02	14.30	0.62	Rejected
6	CVX Chevron	3.48	6.13	0.95	10.85	0.42	Rejected
7	CSCO Cisco	19.00	24.36	1.71	23.13	0.50	Selected

8	KO Coca-Cola	22.20	36.86	0.92	8.19	0.30	Rejected
9	DIS Disney	6.91	7.18	1.07	9.09	0.39	Rejected
10	DWDP DowDuPont Inc	11.03	0.59	1.43	8.34	0.24	Rejected
11	XOM Exxon Mobil	1.97	6.01	0.52	16.91	0.73	Rejected
12	GS Goldman Sachs	25.88	14.07	0.83	2.27	0.04	Rejected
13	HD Home Depot	9.72	26.44	0.43	13.82	2.14	Rejected
14	IBM	10.62	29.92	0.97	7.91	0.44	Rejected
15	INTC Intel	26.37	23.95	1.45	40.15	0.51	Selected
16	JNJ Johnson & Johnson	19.49	24.24	1.07	88.48	0.51	Selected
17	JPM JPMorgan Chase	31.13	14.20	0.89	5.29	0.04	Rejected
18	MCD McDonald's	28.27	-10.87	1.20	7.73	0.46	Rejected
19	MRK Merck	20.60	24.00	0.96	11.16	0.47	Rejected
20	MSFT Microsoft	30.41	36.64	2.57	21.17	0.46	Rejected
21	NKE Nike	9.69	37.18	1.85	53.98	1.36	Rejected
22	PFE Pfizer	29.04	17.93	1.06	9.20	0.33	Rejected
23	PG Procter & Gamble	15.45	23.52	0.55	31.88	0.60	Rejected
24	TRV Travelers Companies Inc	8.93	10.76	0.30	11.36	0.28	Rejected
25	Raytheon Technologies Corporation	5.32	3.99	1.01	2.89	0.37	Rejected
26	UNH UnitedHealth	5.83	23.23	0.74	12.11	1.37	Rejected
27	VZ Verizon	15.30	30.89	0.93	7.36	0.42	Rejected
28	V Visa	49.90	35.92	1.75	27.73	0.29	Rejected
29	WMT Wal-Mart	2.27	14.91	0.24	9.04	2.29	Rejected
30	WBA Walgreen	2.14	9.53	0.74	4.81	1.65	Rejected

*Source: macrotrends and gurufocus.com

Only five securities namely; Apple (AAPL), Cisco Systems (CSCO), Intel (INTC), Johnson & Johnson (JNJ), and 3M (MMM) meet all the parameters such as NPM more than or equal to 10%, ROE more than or equal to 10%, LR more than or equal to 1, ICR more than or equal to 10%, and ATR more than or equal to 0.50 and included in construction of an optimum portfolio, as shown in Table 1.

5.   DATA ANALYSIS

Table2:Shows the Return and Risk of selected securities

Sl. No.	Companies	Return	Risk	Coefficient of variation (CV)
1	AAPL	0.377	0.718	1.907
2	CSCO	0.312	0.593	1.903
3	INTC	0.218	0.457	2.098
4	JNJ	0.139	0.220	1.577
5	3M	0.119	0.227	1.912
DJIA Index		**0.097**	**0.179**	**1.837**

*Source: Authors' computation

It can be observed from Table-2 that Apple has higher average return of 38 percent, followed by Cisco with 31 percent, Intel with 22 percent, JNJ with 14 percent and 3M with 12 percent.However, there is no consistency among the selected securities.

Harry Markowitz Model to construct portfolio return and risk:

Portfolio Expected Return

$$E(R_p) = W(R_1) + W(R_2) + W(R_3) + W(R_4) + W(R_5) \qquad \text{……... (1)}$$

Portfolio risk:

The following is a general formula for determining the optimum weights of two securities, X and Y, in order to minimise portfolio variance:

$$W = \frac{\sigma_y^2 \quad Cov_{xy}}{\sigma_x^2 + \sigma_y^2 \quad 2Cov_{xy}} \qquad \text{……... (2)}$$

Where W* is the optimum portion of investment in security X Investment in Y will be (1-W)

The total risk of a portfolio made up of five assets:

$$\sigma_p^2 = (w_1\sigma_2)^2 + (w_2\sigma_2)^2 + (w_3\sigma_3)^2 + (w_4\sigma_4)^2 + (w_5\sigma_5)^2 + 2w_1w_2(Cov_{12})$$
$$+ 2w_1w_3(Cov_{13}) + 2w_1w_4(Cov_{14}) + 2w_1w_5(Cov_{15}) + 2w_2w_3(Cov_{23})$$
$$+ 2w_2w_4(Cov_{24}) + 2w_2w_5(Cov_{25}) + 2w_3w_4(Cov_{34}) + 2w_3w_5(Cov_{35})$$
$$+2w_4w_5(Cov_{45}) \qquad \text{……... (2.1)}$$

Table3 : Shows the optimum portfolio under Harry Markowitz Model

Sl. No.	Companies	Return		Risk		Optimum Portfolio	
		X_i	R_p	X_i	σ_p	X_i	CV
1	AAPL	0.96		0.01		0.01	
2	CSCO	0.01		0.01		0.01	
3	INTC	0.01	0.370	0.01	0.030	0.01	0.220
4	JNJ	0.01		0.52		0.55	
5	3M	0.01		0.45		0.42	

*Note: Xi: weightage, Rp: portfolio return,?p: portfolio risk and CV: Coefficient of variation

The highest proportion of investment (96%) is to be made in AAPL, which has the highest weighted average rate of return. However, in terms of lowest weighted average rate of risk1/2 and 4/9 of total investments are to be made in JNJ and 3M. The maximum consistency at the optimum weightage reveals that only two stocks; JNJ, and 3M, make up the perfect portfolio with optimum investment proportions of 55 percent and 42 percent,

respectively.

Sharpe's Single Index Model to construct portfolio return and risk:

1. **Portfolio Return:**

$$E(R_i) = \alpha + R_m \, \beta + e_i \qquad\qquad \dots\dots (3)$$

where, R_i is the return of asset, ?is alpha, ?is beta, Rm the market return, ei is random residual error.

2. **Portfolio Risk:**

Systematic Risk $= \beta_i^2 * \sigma_m^2$

Unsystematic Risk $(\sigma_{ei}^2) = \sigma_i^2 - \beta_i^2 * \sigma_m^2$

The steps in constructing optimum portfolio are as follows:

a)   Calculation of return, risk & beta of selected companies.

b)   Calculation of excess return to beta ratio for each stock.

c)   Ranking them from highest to lowest.

d)   Finally, proceed to calculate Ci for all the stocks by using thefollowing formula:

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i \quad R_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

e)   The cumulative values of Ci begin decreasing after a particular Ci, and that point is considered the cut-off point and that stock ratio is the cut-off ratio.

f)   Once the securities for portfolio are chosen, the percentage of funds to be invested in each security can be estimated as follows.

$$X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$$

$$Z_i = \left(\frac{R_i \quad R_f}{\beta_i} \quad C \right) \frac{\beta_i}{\sigma_{ei}^2}$$

g)   C* = Cut off point

h)   Calculations of portfolio return:

$$R_p = \alpha_i + \beta_i R_m$$

i)   Calculations of portfolio variance:

$$\sigma_p^2 = [(\sum_{i=1}^{n} x_i \beta_i)^2 \sigma_m^2] + [\sum_{i=1}^{n} x_i^2 \sigma_{ei}^2]$$

j)   Xi = Stock weightage in the portfolio

Table4:Shows the systematic and unsystematic risk of sample

Sl. No.	Companies	Beta (β_i)	Systematic Risk ($\beta_i^2 \, \sigma_m^2$)	Unsystematic Risk ($\sigma_{ei}^2 = \sigma_i^2 \, \beta_i^2 \, \sigma_m^2$)
1	AAPL	1.236	0.049	0.467
2	CSCO	1.688	0.091	0.261
3	INTC	1.123	0.040	0.168
4	JNJ	0.525	0.009	0.039
5	3M	0.831	0.022	0.029

*Source: Authors' computation

Table4 shows that stocks having higher beta values have high systematic risks, and vice versa. Unsystematic risk mainly depends on variances of respective stocks.

Table5:Shows the Ranking of stocks based on Excess return to beta ratio

Sl. No.	Companies	$R_i \, R_f$	$\dfrac{R_i \, R_f}{\beta_i}$	Rank
1	AAPL	0.330	0.267	1
2	CSCO	0.265	0.157	3
3	INTC	0.171	0.152	4
4	JNJ	0.093	0.177	2
5	3M	0.072	0.087	5

*Note: Ri: stock return, and Rf: risk free rate of return

It can be observed from Table5 that Apple company has 0.267, higher excess return to beta ratio followed by JNJ with 0.177, Cisco with 0.157, Intel with 0.152 and 3M with 0.087 respectively. Interestingly, Apple company ranked top both in excess return and excess return to beta ratio.

Table6:Cut-off point

Sl. No.	Companies	$\dfrac{(R_i \, R_f)\beta}{\sigma_{ei}^2}$	$\dfrac{\beta_i^2}{\sigma_{ei}^2}$	$\sum_{t=1}^{J} \dfrac{(R_i \, R_f)}{\sigma_{ei}^2}$	$\sum \dfrac{\beta_i^2}{\sigma_{ei}^2}$	C*
1	AAPL	0.874	3.270	0.874	3.270	0.025
2	JNJ	1.240	6.997	2.114	10.266	0.051
3	CSCO	1.717	10.916	3.831	21.182	0.073
4	INTC	1.145	7.507	4.975	28.689	0.083
5	3M	2.045	23.453	7.020	52.142	0.084

Note: C: cut-off point

While calculating C* as per Sharpe model, all five securities namely; Apple (AAPL), Johnson & Johnson (JNJ), Cisco System (CSCO), Intel (INTC), and 3M (MMM) are selected for building Optimum Portfolio Construction. The C* goes on, increases from 0.025 to 0.084.

Table7:Shows optimum portfolio under Sharpe's Single Index Model

Sl. No.	Companies	$\dfrac{R_i\ R_f}{\beta_i}$	$\dfrac{R_i\ R_f}{\beta_i}\ C$	$\dfrac{\beta_i}{\sigma_{ei}^2}$	Z	X_i
1	AAPL	0.267	-0.183	2.646	-0.069	0.708
2	JNJ	0.177	-0.093	13.316	-0.007	0.071
3	CSCO	0.157	-0.073	6.468	-0.011	0.116
4	INTC	0.152	-0.068	6.684	-0.010	0.104
5	3M	0.087	-0.003	28.231	0.000	0.001

*Source: Authors' computation

Xi specifies the percentage of funds to be invested in each of these chosen securities. Nearly 71% of funds have to be invested in Apple (AAPL), 12% in Cisco Systems (CSCO), 10% in Intel (INTC), 7% in Johnson & Johnson (JNJ) and0.1%, lowest weightage in 3M (MMM).

Table8:Shows optimum portfolio return, risk and variance

Sl. No.	Companies	X_i	α_i	β_i	R_m	Expected portfolio return
1	AAPL	0.708	0.181	0.875	0.097	0.267
2	CSCO	0.071	0.017	0.195	0.097	0.036
3	INTC	0.116	0.011	0.117	0.097	0.023
4	JNJ	0.104	0.006	0.037	0.097	0.010
5	3M	0.001	0.000	0.001	0.097	0.000
Expected portfolio return						0.335
Portfolio Variance (σ_p^2)						0.287
Portfolio Risk (σ_p)						0.536

*Source: Authors' computation

Apple expects 26.7%, the highest rate of return followed by the rest in the portfolio not more than 4%. The portfolio returns sums up to 33.5%. Expected portfolio variance and standard deviation (risk) are 28.7 and 53.6% respectively.

Table-9:Shows the comparative resultsof Harry Markowitz and Sharpe's Single Index model

Sl. No.	Companies	X_i		R_p		σ_p		CV	
		HMM	SIM	HMM	SIM	HMM	SIM	HMM	SIM
1	AAPL	0.01	0.71						
2	CSCO	0.01	0.12						
3	INTC	0.01	0.10	0.370	0.335	0.030	0.536	0.081	0.857
4	JNJ	0.55	0.07						
5	3M	0.42	0.00						

*Note: HMM: Harry Markowitz Model, SIM: Single Index Model, Xi: weightage, Rp: portfolio return, ?p: portfolio risk and CV: Coefficient of variation.

On comparison between two models,the portfolio expected returns are almost the same. However, there is greater and wider variation in the portfolio risk. Despite the coefficient of variation is less than one in both the models, HMM is more consistent than that of SIM.

6.   CONCLUSION

Optimum portfolio creation is critical and challenging for both institutional and individual investors. This research aimed to build and compare the optimum portfolios of Harry Markowitz and Sharpe's Single Index models. Only five securities viz., Apple, Cisco, Intel, Johnson & Johnson, and 3M were selected for construction of optimum portfolio. Under Harry Markowitz model with varying proportion of investments incorporating the maximum of 55 percent of total investment in JNJ, followed by 42 percent in 3M and a negligible percentage in other securities.Similarly, Sharpe's single index model for optimum portfolio shows that almost 5/7 proportion of total investment should be invested in Apple, followed by 1/8 in Cisco, 10% in Intel and negligible percentage of other stocks.The portfolio expected returns are almost same between two models with the greater wider variation in the portfolio risk. Despite the coefficient of variation is less than one in both the models, HMM is more consistent than that of SIM.

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